Diagnosing The Conditional Dependence Between Returns And Risk With Vector Autoregressive Model During Covid Crisis

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Abstract

Introduction: This paper proposed two separate tests for checking the conditional dependence between returns and risk of selected securities using Vector Autoregressive (VAR) model.

Methodology: The proposed first test is based on Special Wald's F-statistic. This test was employed in order to check whether the expected returns conditionally depend on risk and past year returns if the returns follow normal distribution. Similarly, in order to scrutinize the conditional dependence of risk on return and past years risk, the second test based on Lagrange's multiplier (LM) statistic was employed. The methodology consists to model the data over the security returns of selected 5 companies under FMCG Industry listed in National Stock Exchange (NSE), India over the period between Jan 1, 2020 to Dec 31, 2020.

Results: From the result of the study, it is revealed that even though the stock liquidity of Britannia and Marico is good, their expected returns reveals that this was not a deciding factor on their past year risk and return during the period of the study. And also Nestle and ITC proved to be their risk has an influence over their past risk and past returns.

Keywords: returns, risk, conditional dependence, autoregression, vector autoregression, heteroscedasticity, Special Wald test, Lagrange's multiplier test

Introduction and related works

Vector autoregression (VAR) is an econometric model and widely used in econometric applications. It is said to be a time series model in which it is used when two or more time series have an impact on each other and known to be inter dependence between multiple time series. The reason that VAR is considered as an Autoregressive model (AR), it is proven for forecasting. Therefore, with the help of the Past actions or behaviours of variables it is modelled as a function. Following are the works regarding the studies especially on Vector autoregression by adopting new framework. Sims (1980) first proposed Vector Auto Regression (VAR). Phillips and Hansen (1990) designed Fully modified least squares (FM – OLS) regression. Philips (1995) developed an asymptotic theory based on the above method and also with Fully modified Vector autoregression (FM-VAR). The fully modified (FM) estimators designed to estimate cointegrating relations directly by modifying traditional OLS. Le et.al (1996) introduced Univariate mixture autoregressive models and it was further developed by Wong and Li (2000), in order to modelling the non-linear time series and they introduced mixture autoregressive (MAR) model by generalizing Guassian mixture transition distribution (GMTD) model. This model assists to modelling the time series along with Multimodal conditional distributions and with heteroscedasticity. Balcilar et.al (2016) used non-linear logistic smooth transition vector autoregressive model (LSTVAR) and analyzed whether the financial shocks especially the positive and negative shocks have asymmetric effect or not. Kalliovirta et.al (2016) developed Gaussian mixture vector autoregressive (GMVAR) model, which is a new nonlinear vector autoregressive (VAR) model and a mixture VAR model (MVAR) which focus on regime switching behavior. Qian (2016) studied a VAR model with varied frequency data in a Bayesian context. Lower frequency (aggregated) data are essentially a linear combination of higher frequency (disaggregated) data. Luintel and Khan (1999) examined the long-run relationship between financial development and economic growth in a MVAR. Ahalawat and Patro (2019) Predicted the predict dynamic behaviour of economic and financial time series The outcomes of vector autoregression approach depicted that the two variables have positive impact and are statistically significant in the short run. Christopoulos and Tsionas (2003) investigated the long run relationship between financial depth and economic growth, trying to utilize the data in the most efficient manner via panel unit root tests and panel cointegration analysis. The long run relationship is estimated using fully modified OLS. Rahbek and Nielsen (2014) proposed a discrete-time multivariate model where lagged levels of the process enter both the conditional mean and the conditional variance. Specifically, the model bridges vector autoregressions and multivariate ARCH models in which residuals are replaced by levels lagged. The model thus allows for volatility induced stationarity and the paper shows conditions under which the multivariate process is strictly stationary and geometrically ergodic. This paper utilised the Vector autoregressive model and proposed a test to check the conditional dependency between return and risk based on Special Wald and Lagrange's multiplier test statistic. The proposed tests were performed based on two assumptions namely

Expected returns are Non-Stationary.

Risk of the security is Heteroscedastic

- **Section 2 - Testing the conditional dependency of returns on risk**

Based on the above pointed assumptions, the authors made an attempt to test the following Vector autoregression of the expected returns on risk is given as follows:

$$E\left(X_t / \sigma^2_{X_t}, \sigma^2_{X_{t-1}}, \ldots, \sigma^2_{X_{t-p-1}}, \sigma^2_{X_{t-p}}, \mu_{X_{t-1}}, \ldots, \mu_{X_{t-q-1}}, \mu_{X_{t-q}} \right) = \alpha + \sum_{i=0}^{p} \alpha_{1i}\sigma^2_{X_{t-i}} + \sum_{j=1}^{q} \alpha_{2j}\mu_{X_{t-j}}$$

-(2.1)

Vector autoregressive model*(Var)* of lag length *(p,q)* is used to perform the test based on the above said assumptions, the following steps need to adopt:

Step 1: Consider the actual returns*(X_t)* as an endogenous variable and the Squared deviations from the average returns namely $(X_t - \overline{X})^2, (X_{t-1} - \overline{X})^2, \ldots, (X_{t-p-1} - \overline{X})^2, (X_{t-p} - \overline{X})^2$ $X_{t-1}, X_{t-2}, X_{t-3} \ldots, X_{t-q-1}, X_{t-q}$ are exogenous variables.

Step 2: Based on (2.1), Regress the actual returns of the security at time 't' with the lagged Squared deviations from the average returns and the lagged actual returns of *(p,q)*the security with a lag length of by using the method of OLS for (2.2) and $u_t \sim ND\left(0, \sigma^2_u\right)$

which given as $X_t = \alpha + \sum_{i=0}^{p} \alpha_{1i}\left(X_{t-i} - \overline{X}\right)^2 + \sum_{j=1}^{q} \alpha_{2j}X_{t-j} + u_t$

Step 3: Compute the Wald's *F*-statistic $F' = \frac{R^2}{K-1} / \frac{1-R^2}{n-K} \sim F(K-1, n-K)$

which follows *F*-distribution with (*K*-1, n-*K*) degrees of freedom,k=p+q+1 is the estimated no. of parameters which is equal to the no. of lags used plus one and R^2 is the unadjusted R-squared from the regression model (2.2).

Step 4: Reject the null hypothesis that

$$H_0 : \alpha_0 = \alpha_{10} = \alpha_{11} = \alpha_{12} = \cdots = \alpha_{1(p-1)} = \alpha_{1p} = \alpha_{21} = \alpha_{22} = \cdots = \alpha_{2(q-1)} = \alpha_{2q} == 0$$

if the computed $F' > F_{0.05}(K-1, n-K)$ o r $F' > F_{0.01}(K-1, n-K)$ the upper Percent point o n the F-distribution with (*K*-1, n-*K*) degrees of freedom at 5% and 1% significance level respectively.

Step 5: Decision - The rejection of the null hypothesis confirms that the expected return of the security conditionally depends on the risk and returns over different time periods. If the acceptance proves, that the expected returns is equal to the constant , this shows that the expected returns is constant not depends on the risk and returns over time. Similarly, the risk and returns over different time periods are not having any effect on its actual returns at time 't'.

- **Section 3 - Testing the conditional dependency of risk on returns**

In this section, another attempt is made to test the following Vector autoregression of the risk on the expected returns of the security is given as follows:

$$E\left((X_t-\mu X)^2 / \mu_{X_t},\mu_{X_{t-1}},\dots,\mu_{X_{t-r-1}},\mu_{X_{t-r}},\sigma^2_{X_{t-1}},\dots,\sigma^2_{X_{t-s-1}},\sigma^2_{X_{t-s}}\right)=\beta+\sum_{i=0}^{r}\beta_{1i}\mu_{X_{t-i}}+\sum_{j=1}^{s}\beta_{2j}\sigma^2_{X_{t-j}}$$

- (3.1)

Vector autoregressive model*(Var)* of lag length *(r,s)* is used to apply the test based on the above assumptions, the following steps need to adopt:

Step 1: Consider the Squared deviation from the average returns $\left(X_t-\overline{X}\right)^2$ and time 't'as an endogenous variable and the lagged actual returns namely $X_{t-1},X_{t-2},X_{t-3}\dots,X_{t-r-1},X_{t-r}$ and $(X_{t-1}-\overline{X})^2,(X_{t-2}-\overline{X})^2\dots,(X_{t-s-1}-\overline{X})^2,(X_{t-s}-\overline{X})^2$ are exogenous variables.

Step 2: From (3.1), Regress the lagged values of the security returns with the squared deviation from the average returns by using the method of OLS for (2.4).

$$\left(X_t-\overline{X}\right)^2=\beta+\sum_{i=0}^{r}\beta_{1i}X_{t-i}+\sum_{j=1}^{s}\beta_{2j}\left(X_{t-j}-\overline{X}\right)^2+v_t$$

- (3.2)

Step 3: Since is non-normally distributed, then perform the LM test by computing the chi-square-statistic $nR^2 \sim CHISQ(K)$ which follows Chi-Square distribution with *K* degrees of freedom, where *K=r+s+1*is the estimated no. of parameters which is equal to the no. of lags used plus one and is the unadjusted R-squared from the regression model (3.2)

Step 4 : Reject the null hypothesis that ,

$$H_0:\beta_0=\beta_{10}=\beta_{11}=\beta_{12}=\dots=\beta_{1(r-1)}=\beta_{1r}=\dots=\beta_{21}=\beta_{22}=\dots=\beta_{2(s-1)}=\beta_{2s}=0$$

if the computed $CHISQ\ statistic' > CHISQ_{0.05}(K)$ or , $CHISQ\ statistic' > CHISQ_{0.01}(K)$ the upper Percent point of the Chi-square distribution with *K* degrees of freedom at 5% and 1% significance level respectively.

Step 5 Decision: The rejection of the null hypothesis confirms that risk of the security conditionally depends on the expected returns over different time periods and the risk of the security in the past time periods. If the acceptance confirms, that the risk is equal to the constant and this shows that risk is not depending on its security returns and risk over time. Similarly, the expected returns and risk of the securities over different time periods has no influence on its risk at time 't'.

- **Section 4 - Data and methodology**

This section deals with the data and methodology for the proposed test by selecting equity securities from FMCG Industry listed in National Stock Exchange (NSE), India. The selected securities from FMCG Industry are ITC, Britannia, Dabur, Marico and Nestle. The historical prices of these securities were collected over the period from January 1, 2020 to December 31, 2020 with 252 Observation and its returns are calculated. The proposed tests were performed with the help of the Gretl version 2020d (Gnu Regression, Econometrics and Time-series library). The test results are presented from Table-1 to Table-10 in which testing the conditional dependence of present year expected returns on past year risk and expected returns of the selected securities are shown in Table-1 to Table-5. Likewise, testing the conditional dependence of present year risk on past year expected returns and risk of selected securities are visualized from Table 6 to Table 10.

- **Section 5 - Discussion**

Table 1-5 visualizes the results of the Special Wald test based on F-statistic of the selected securities of FMCG Industry listed in NSE, India. The test was conducted to scrutinize the Vector autoregression of expected returns on risk with securities returns as dependent variable and risk as independent variable with 30 combinations. The results of the analysis exhibits that most of the securities attained the significance level at 5% and 1% level in which it shows that expected returns conditionally dependent on risk as well as the past year expected returns. Securities such as Britannia, Dabur and Marico do not achieve the level of significance at 5% and 1% respectively in several lag combinations. Britannia does not achieve the level of significance in (0,1) (0,2) (0,3) (0,4) and (0,5) lag periods and this exhibits that the null hypothesis is accepted in these periods. This shows that there is no conditional dependence between present year expected returns on Past year risk and past year expected returns in these specific periods. Regarding Marico, in (1,1) and (1,3) does not achieve the level of significance hence the risk is proven to be heteroscedastic. This shows that there is no conditional dependence between present year expected returns on Past year risk and past year expected returns in these specific periods for Marico. Regarding Dabur, all the 30 combinations of lag periods does not achieve its level of significance. Hence it is proved that from (0,1) to (5,5) periods there is no conditional dependence between present year expected returns on Past year risk and past year expected returns in these specific periods for Dabur. This shows that the expected returns are constant not depends on the risk and returns over time. Similarly, table-6-10 exhibits the results of Lagrange's multiplier test of the securities of the selected FMCG securities. The test was conducted to scrutinize the Vector autoregression of risk on expected returns with risk as independent variable and securities returns as 5 independent lagged variable periods with 30 combinations. The results revealed that for the majority of the securities their risk is conditionally dependent on their expected returns. Nestle does not achieve the level of significance for the entire lag time periods. Whereas, Britannia and Marico achieved the level of significance in certain periods. Therefore, it is proved that the null hypothesis is accepted and with respect to Nestle there is conditional dependence of present year risk on past year expected returns and risk over the entire time period considered in the study. At the same time regarding Britannia, apart from (0,1) and (0,2) lag periods all the other combinations from (0,3) to (5,5) not achieved the level of significance. Hence it is proved that there is conditional dependence of past year's risk exists. Concerning Marico, the result of the analysis reveals that the lag periods such as (0,1), (0,2) (0,3) (0,4) (0,5) and (1,1) are significant than the following periods from (1,2) to (5,5). Hence it shows that the risk of the above said securities have no significant relationship with the returns in the remaining periods. and it is proved that there is no conditional dependence of risk on returns for Marico in these specified time period.

Conclusion

From the above statistical inference, the following considerations were made: In this study the authors diagnosed conditional dependence between returns and risk by using vector autoregressive model. The authors emphasized on the dependency and independency of security returns, not of the lags. Here, in this study lags are taken arbitrarily. In Vector autoregressive models, the time series must influence each other variables and as a statistical model it is used to find out the influence on the endogenous variables, especially how it changes over time. Therefore, the results of the study shows that if the returns have influence over risk, then it is said to be heteroscedastic and if returns have its effect on its previous year returns then it is non-stationary in nature. Securities under FMCG Industry listed in NSE, India were considered in the study. Security returns other than Britannia, HCL and Dabur has no significant effect on their previous year returns. And also, it is revealed that ITC and Nestle have no relationship either their previous year returns nor their risk. Security returns such as Britannia and Marico have an effect and said to be Jointly conditional with its previous year's return and risk. Regarding the risk of these securities, for the specified time period it is proven that the null hypothesis is rejected and alternative hypothesis is accepted, Hence the risk of the security is Heteroscedastic. Whereas security returns of Dabur have no conditional dependent on its previous year returns. Hence it is revealed that it is asymmetric in nature in those time period. Based on the vector auto-regressive model the results of the analysis varies and changes over different time period.

Whenever the market declines, the trend of FMCG stocks in Nifty are in high demand usually for a past decade. Its demand based on several factors other than its risk aspect. In this study Stocks such as Britannia and Marico are proved that even though their stock liquidity their expected returns was not a deciding factor on their past year risk and return. Moreover, Britannia, Dabur and Marico's risk may influence its expected return. Therefore, the investors must look into the various factors of these individual stocks in order to minimize the risk in the future. Especially, the challenge is for the investors during this pandemic period is whether to build a new portfolio or to reframe it with other sectorial stocks because of the setback in FMCG stocks. The risk of Nestle and ITC proved that it has an influence over their past risk and past returns. FMCG Sector in both Recession and in boom must be highly considered by the investors that what were their risk factors. That may have an impact on the following years.

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Table 1. Testing the conditional dependence of present year expected returns on past year risk and expected returns of ITC

Lags		Con-stant	$\sigma^2_{X_t}$	$\sigma^2_{X_{t-1}}$	$\sigma^2_{X_{t-2}}$	$\sigma^2_{X_{t-3}}$	$\sigma^2_{X_{t-4}}$	$\sigma^2_{X_{t-5}}$	Special Wald Test	
p	q	α	α_{11}	α_{12}	α_{13}	α_{14}	α_{15}	α_{16}	(*df*1, *df*2)	F-statistic
0	1	0.0014		58.3886	-	-	-	-	(2, 248)	4.249771**
0	2	0.0019		135.8250		-	-	-	(3, 246)	3.617209**
0	3	0.0008				951.891**	-	-	(4, 244)	6.437213**
0	4	0.0003				782.790**	907.149**	-	(5, 242)	8.129322**
0	5					646.457**	812.718**	544.149*	(6, 240)	7.563860**
1	1	0.0014		30.9075	-	-	-	-	(3, 247)	2.891407*
1	2	0.0019		104.8540		-	-	-	(4, 245)	2.774619*
1	3	0.0008				948.232**	-	-	(5, 243)	5.132786**
1	4					742.769**	970.245**		(6, 241)	6.962970**
1	5					576.653*	887.495**	605.403*	(7, 239)	6.798953**
2	1	0.0016			-	-	-	-	(4, 245)	2.956150*
2	2	0.0020		29.7861		-	-	-	(5, 244)	2.634616*
2	3	0.0008				972.774**	-	-	(6, 242)	4.781186*
2	4					761.839**	1023.92**		(7, 240)	6.711385**
2	5					620.378*	944.4130	501.8990	(2, 248)	4.249771**
3	1	0.0016		0.559703	-	-	-	-	(8, 238)	6.338591**
3	2	0.0019		49.7772		-	-	-	(5, 243)	2.735956**
3	3	0.00078				955.956**	-	-	(6, 242)	2.417619*
3	4					766.130	1025.59**		(7, 241)	4.092835**
3	5					632.264**	948.740**	507.700	(8, 239)	5.848860**
4	1	0.00173			-	-	-	-	(9, 237)	5.618413*

4	2	0.00198		10.7799		-	-	-	(6, 241)	2.541522*
4	3	0.00075				1047.33	-	-	(7, 240)	2.263914*
4	4					828.622	964.347		(8, 239)	4.056330**
4	5					691.321**	898.605**	488.897	(9, 238)	5.317510**
5	1	0.00147			-	-	-	-	(10, 236)	5.131058**
5	2	0.00164				-	-	-	(7, 239)	5.401373**
5	3	0.00058				924.775**	-	-	(8, 238)	4.749799**
5	4					768.420**	768.205**	-	(9, 237)	5.859736**
5	5					565.468*	648.799*	692.832	(10, 236)	6.314250**

n=251 *Significant at 5% level **Significant at 1% level

Table 2. Testing the conditional dependence of present year expected returns on past year risk and expected returns of Britannia

Lags		Con-stant	$\sigma^2_{X_t}$	$\sigma^2_{X_{t-1}}$	$\sigma^2_{X_{t-2}}$	$\sigma^2_{X_{t-3}}$	$\sigma^2_{X_{t-4}}$	$\sigma^2_{X_{t-5}}$	$\overline{X}_{t-1}$	Special Wald test	
p	q									**(*df*1, *df*2)**	**F-statistic**
0	1	0.00044		381.15	-	-	-	-	-	(2, 248)	1.297681
0	2	0.000007		217.76	415.30*	-	-	-	-	(3, 246)	1.890985
0	3			220.51	285.67	353.56	-	-	-	(4, 244)	1.923963
0	4	-0.000032		321.01	282.24	474.72*		-	-	(5, 242)	1.817577
0	5	-0.000014		319.98	284.84	474.63*			-	(6, 240)	1.504839
1	1	0.00070		413.66	-	-	-	-		(3, 247)	4.045866**
1	2	0.00017		218.29	503.04*	-	-	-		(4, 245)	4.223773**
1	3			222.32	333.64	484.24	-	-		(5, 243)	4.183683**
1	4	0.00008		302.07	329.84	578.05		-		(6, 241)	3.630139**
1	5	0.00011		290.76	351.05	575.82				(7, 239)	3.094763**
2	1	0.00053		415.16	-	-	-	-		(4, 245)	3.223854**
2	2	0.00007		224.76	488.35*	-	-	-		(5, 244)	3.474815**
2	3			225.13	332.64	467.73	-	-		(6, 242)	3.490924**
2	4	0.00004		312.02	328.12	560.44		-		(7, 240)	3.133275**
2	5	0.00008		299.49	352.54	556.88				(2, 248)	2.729862**
3	1	0.00021		479.42*	-	-	-	-		(8, 238)	3.342416**

3	2			289.13	488.52*	-	-	-		(5, 243)	3.546861**
3	3			283.56	349.58	418.60*	-	-		(6, 242)	3.460665**
3	4			412.59	346.16	538.76*		-		(7, 241)	3.275091**
3	5			390.20	399.52	530.05*				(8, 239)	2.932958**
4	1	0.00020		486.61	-	-	-	-		(9, 237)	2.782251**
4	2			297.57	509.82*	-	-	-		(6, 241)	3.075363**
4	3			292.32	371.21	420.06	-	-		(7, 240)	3.059432**
4	4			430.83	373.12	547.75*		-		(8, 239)	2.971733**
4	5			405.34	448.48	538.28*				(9, 238)	2.720162**
5	1	0.00018		489.13*	-	-	-	-		(10, 236)	2.365667*
5	2			301.58	521.12*	-	-	-		(7, 239)	2.687036**
5	3			299.06	381.85	449.89	-	-		(8, 238)	2.751837**
5	4			436.57	383.45	576.07*		-		(9, 237)	2.699522**
5	5			409.38	466.42	569.26*				(10, 236)	2.516796**

n=251 *Significant at 5% level **Significant at 1% level

Note: Above two tables are abridged versions of original large tables. Table-3 on Dabur, Table-4 on Marico, Table-5 on Nestle, Table-6 on ITC Britannia and some other very large but useful tables could not be placed here with this article, because of space issues. *We request all readers and interested scholars to contact the author for complete tables, for their reference. We apologize for the inconvenience.*